پاسخ تمرین های تکلیف شماره۲
درس مخابرات ۲

محمّدعلی شفیعیان

پاسخ تمرین ۱ :

(الف)

To show that the waveforms $\psi_n(t)$, $n = 1, \ldots, 3$ are orthogonal we have to prove that

$$\int_{-\infty}^{\infty} \psi_m(t)\psi_n(t)dt = 0, \qquad m \neq n$$

Clearly,

$$\begin{aligned} c_{12} &= \int_{-\infty}^{\infty} \psi_1(t)\psi_2(t)dt = \int_0^4 \psi_1(t)\psi_2(t)dt \\ &= \int_0^2 \psi_1(t)\psi_2(t)dt + \int_2^4 \psi_1(t)\psi_2(t)dt \\ &= \frac{1}{4}\int_0^2 dt - \frac{1}{4}\int_2^4 dt = \frac{1}{4} \times 2 - \frac{1}{4} \times (4-2) \\ &= 0 \end{aligned}$$

Similarly,

$$\begin{aligned} c_{13} &= \int_{-\infty}^{\infty} \psi_1(t)\psi_3(t)dt = \int_0^4 \psi_1(t)\psi_3(t)dt \\ &= \frac{1}{4}\int_0^1 dt - \frac{1}{4}\int_1^2 dt - \frac{1}{4}\int_2^3 dt + \frac{1}{4}\int_3^4 dt \\ &= 0 \end{aligned}$$

and

$$\begin{aligned} c_{23} &= \int_{-\infty}^{\infty} \psi_2(t)\psi_3(t)dt = \int_0^4 \psi_2(t)\psi_3(t)dt \\ &= \frac{1}{4}\int_0^1 dt - \frac{1}{4}\int_1^2 dt + \frac{1}{4}\int_2^3 dt - \frac{1}{4}\int_3^4 dt \\ &= 0 \end{aligned}$$

Thus, the signals $\psi_n(t)$ are orthogonal.

(ب)

We first determine the weighting coefficients

$$x_n = \int_{-\infty}^{\infty} x(t)\psi_n(t)dt, \qquad n = 1, 2, 3$$

$$\begin{aligned} x_1 &= \int_0^4 x(t)\psi_1(t)dt = -\frac{1}{2}\int_0^1 dt + \frac{1}{2}\int_1^2 dt - \frac{1}{2}\int_2^3 dt + \frac{1}{2}\int_3^4 dt = 0 \\ x_2 &= \int_0^4 x(t)\psi_2(t)dt = \frac{1}{2}\int_0^4 x(t)dt = 0 \\ x_3 &= \int_0^4 x(t)\psi_3(t)dt = -\frac{1}{2}\int_0^1 dt - \frac{1}{2}\int_1^2 dt + \frac{1}{2}\int_2^3 dt + \frac{1}{2}\int_3^4 dt = 0 \end{aligned}$$

As it is observed, $x(t)$ is orthogonal to the signal waveforms $\psi_n(t)$, $n = 1, 2, 3$ and thus it can not represented as a linear combination of these functions.

پاسخ تمرین ۲ :

Consider the signals $s_1(t)$, $s_2(t)$, $s_3(t)$, and $s_4(t)$ shown in Fig. 1a. We wish to use the Gram-Schmidt orthogonalization procedure to find an orthonormal basis for this set of signals.

Step 1 We note that the energy of signal $s_1(t)$ is

$$E_1 = \int_0^T s_1^2(t)dt$$

$$= \int_0^{T/3} (1)^2 dt$$

$$= \frac{T}{3}$$

The first basis function $\phi_1(t)$ is therefore

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

$$= \begin{cases} \sqrt{3/T}, & 0 \le t \le T/3 \\ 0 & \text{otherwise} \end{cases}$$

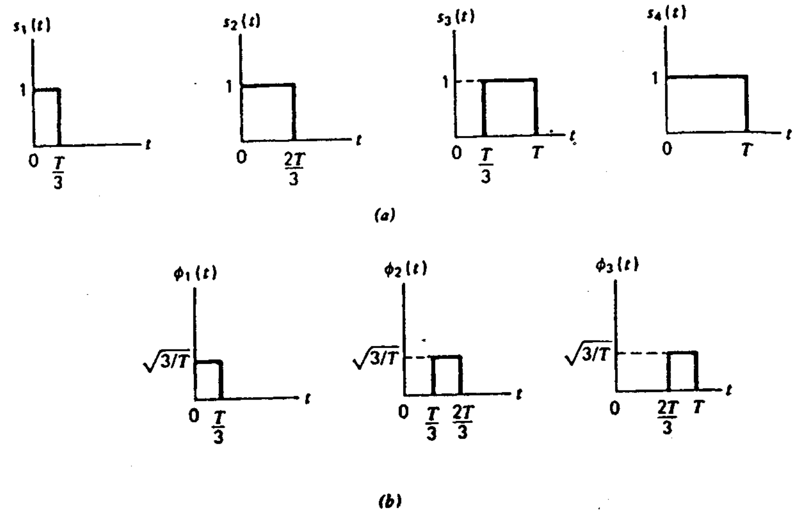


Figure 1

Step 2 Evaluating the projection of $s_2(t)$ onto $\phi_1(t)$, we find that

$$s_{21} = \int_0^T s_2(t)\phi_1(t)dt$$

$$= \int_0^{T/3} (1)\left(\sqrt{\frac{3}{T}}\right)dt$$

$$= \sqrt{\frac{3}{T}}$$

The energy of signal $s_2(t)$ is

$$E_2 = \int_0^T s_2^2(t)$$

$$= \int_0^{2T/3} (1)^2 dt$$

$$= \frac{2T}{3}$$

The second basis function $\phi_2(t)$ is therefore

$$\phi_2(t) = \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 - s_{21}^2}}$$

$$= \begin{cases} \sqrt{3/T}, & T/3 \leq 2T/3 \\ 0 & \text{otherwise} \end{cases}$$

Step 3 Evaluating the projection of $s_3(t)$ onto $\phi_1(t)$,

$$s_{31} = \int_0^T s_3(t)\phi_1(t)dt$$

$$= 0$$

and the coefficient s_{32} equals

$$s_{32} = \int_0^T s_3(t)\phi_1(t)dt$$

$$= \int_{T/3}^{2T/3} (1)\left(\sqrt{\frac{3}{T}}\right)dt$$

$$= \sqrt{\frac{3}{T}}$$

The corresponding value of the intermediate function $g_i(t)$, with $i = 3$, is therefore

$$g_3(t) = s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)$$

$$= \begin{cases} 1, & 2T/3 \leq t \leq T \\ 0, & \text{elsewhere} \end{cases}$$

Hence, the third basis function $\phi_3(t)$ is

$$\phi_3(t) = \frac{g_3(t)}{\sqrt{\int_0^T g_3^2(t)dt}}$$

$$= \begin{cases} \sqrt{3/T}, & 2T/3 \le t \le T \\ 0, & \text{elsewhere} \end{cases}$$

The orthogonalization process is now complete.

The three basis functions $\phi_1(t)$, $\phi_2(t)$, and $\phi_3(t)$ form an orthonormal set, as shown in Fig. 1b. In this example, we thus have $M = 4$ and $N = 3$, which means that the four signals $s_1(t)$, $s_2(t)$, $s_3(t)$, and $s_4(t)$ described in Fig. 1a do not form a linearly independent set. This is readily confirmed by noting that $s_4(t) = s_1(t) + s_3(t)$. Moreover, we note that any of these four signals can be expressed as a linear combination of the three basis functions, which is the essence of the Gram-Schmidt orthogonalization procedure.

پاسخ تمرین ۳ :

The expansion coefficients $\{c_n\}$, that minimize the mean square error, satisfy

$$c_n = \int_{-\infty}^{\infty} x(t)\psi_n(t)dt = \int_0^4 \sin\frac{\pi t}{4}\psi_n(t)dt$$

Hence,

$$\begin{aligned} c_1 &= \int_0^4 \sin\frac{\pi t}{4}\psi_1(t)dt = \frac{1}{2}\int_0^2 \sin\frac{\pi t}{4}dt - \frac{1}{2}\int_2^4 \sin\frac{\pi t}{4}dt \\ &= -\frac{2}{\pi}\cos\frac{\pi t}{4}\Big|_0^2 + \frac{2}{\pi}\cos\frac{\pi t}{4}\Big|_2^4 \\ &= -\frac{2}{\pi}(0-1) + \frac{2}{\pi}(-1-0) = 0 \end{aligned}$$

Similarly,

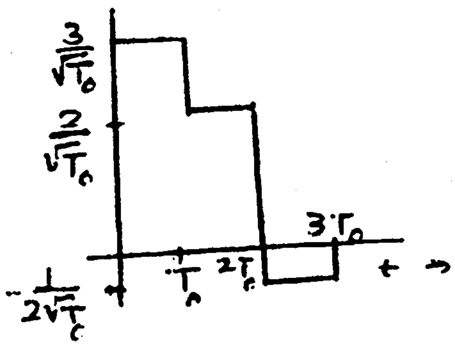
$$\begin{aligned} c_2 &= \int_0^4 \sin\frac{\pi t}{4}\psi_2(t)dt = \frac{1}{2}\int_0^4 \sin\frac{\pi t}{4}dt \\ &= -\frac{2}{\pi}\cos\frac{\pi t}{4}\Big|_0^4 = -\frac{2}{\pi}(-1-1) = \frac{4}{\pi} \end{aligned}$$

and

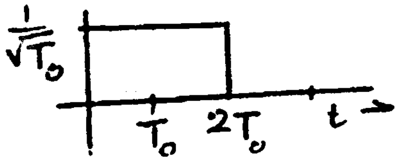
$$\begin{aligned} c_3 &= \int_0^4 \sin\frac{\pi t}{4}\psi_3(t)dt \\ &= \frac{1}{2}\int_0^1 \sin\frac{\pi t}{4}dt - \frac{1}{2}\int_1^2 \sin\frac{\pi t}{4}dt + \frac{1}{2}\int_2^3 \sin\frac{\pi t}{4}dt - \frac{1}{2}\int_3^4 \sin\frac{\pi t}{4}dt \\ &= 0 \end{aligned}$$

Note that c_1, c_2 can be found by inspection since $\sin\frac{\pi t}{4}$ is even with respect to the $x = 2$ axis and $\psi_1(t)$, $\psi_3(t)$ are odd with respect to the same axis.

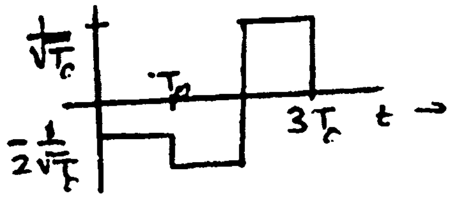
پاسخ تمرین ۴ :



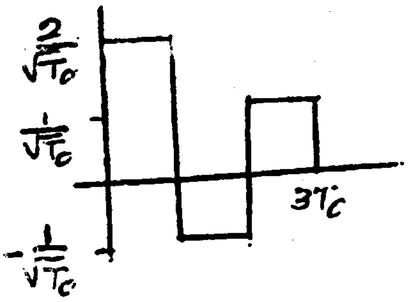
3/√T0
2/√T0
-1/2√T0
T0
2T0
3T0
t



1/√T0
T0
2T0
t



1/√T0
-1/2√T0
T0
3T0
t



2/√T0
1/√T0
-1/√T0
3T0