پاسخ تمرین های تکلیف شماره۳

درس مخابرات ۲

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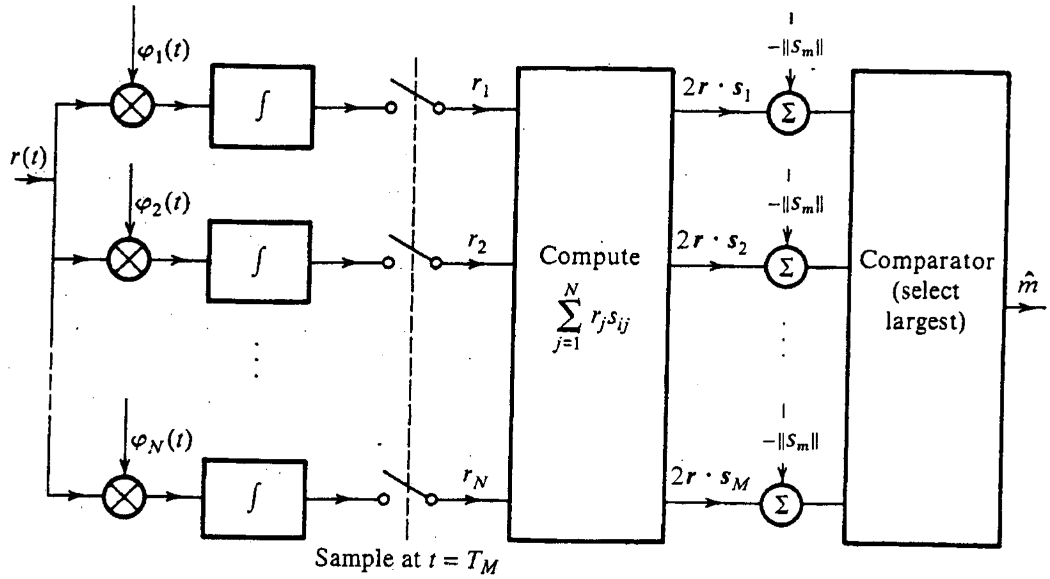
پاسخ تمرین ۱ :



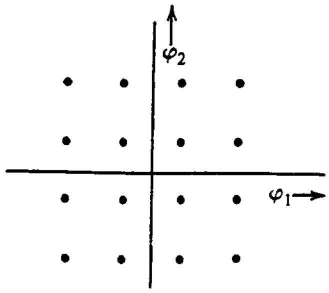
(a)

پاسخ تمرین ۲ :

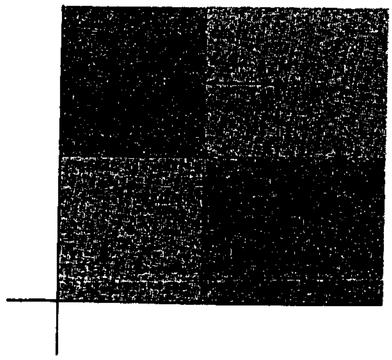
Let us first calculate the error probability. The first quadrant of the signal space is reproduced in Fig. 14.12b. Because all the signals are equiprobable, the decision region boundaries will be perpendicular bisectors joining various signals, as shown in Fig. 14.12b.

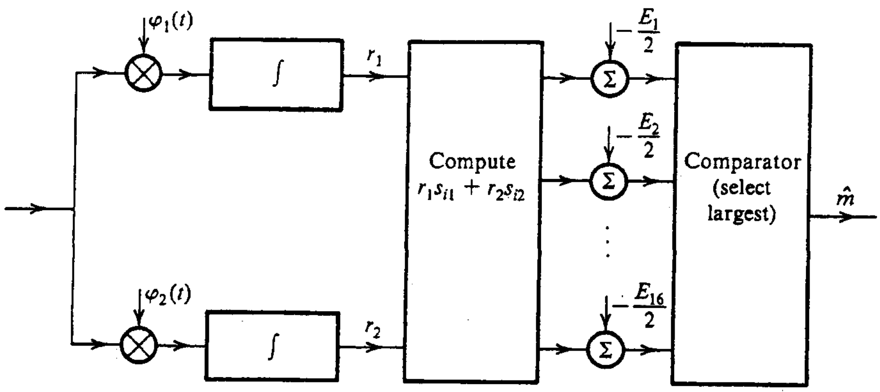
From Fig. 14.12b it follows that

$$
\begin{aligned}
P(C|m_1) &= P(\text{noise vector originating at } s_1 \text{ lies within } R_1)\\
&= P\left(n_1 > -\frac{d}{2},\ n_2 > -\frac{d}{2}\right)\\
&= P\left(n_1 > -\frac{d}{2}\right) P\left(n_2 > -\frac{d}{2}\right)\\
&= \left[1 - Q\left(\frac{d/2}{\sigma_n}\right)\right]^2\\
&= \left[1 - Q\left(\frac{d}{\sqrt{2\mathcal{N}}}\right)\right]^2
\end{aligned}
$$



(a)


(b)



(c)

For convenience, let us define

$$p = 1 - Q\left(\frac{d}{\sqrt{2\mathcal{N}}}\right) \tag{14.55}$$

Hence,

$$P(C|m_1) = p^2$$

Using similar arguments, we have

$$P(C|m_2) = P(C|m_4) = \left[1 - Q\left(\frac{d}{\sqrt{2\mathcal{N}}}\right)\right]\left[1 - 2Q\left(\frac{d}{\sqrt{2\mathcal{N}}}\right)\right]$$
$$= p(2p-1)$$

and

$$P(C|m_3) = (2p-1)^2$$

Because of the symmetry of the signals in all four quadrants, we get similar probabilities for the four signals in each quadrant. Hence,

$$\begin{aligned}
P(C) &= \sum_{i=1}^{16} P(C|m_i)P(m_i) \\
&= \frac{1}{16}\sum_{i=1}^{16} P(C|m_i) \\
&= \frac{1}{16}[4p^2 + 4p(2p-1) + 4p(2p-1) + 4(2p-1)^2] \\
&= \frac{1}{4}[9p^2 - 6p + 1] \\
&= \left(\frac{3p-1}{2}\right)^2
\end{aligned} \tag{14.56a}$$

and

$$P_{eM} = 1 - P(C) = \frac{9}{4}\left(p + \frac{1}{3}\right)(1-p)$$

In practice, $P_{eM} \to 0$, and, hence, $P(C) \to 1$. This means $p \simeq 1$ [see Eq. (14.56a)], and

$$P_{eM} \simeq 3(1-p) = 3Q\left(\frac{d}{\sqrt{2\mathcal{N}}}\right) \tag{14.56b}$$

To express this in terms of the received power S_i, we determine $\bar{E}$, the average energy of the signal set in Fig. 14.12. Because E_k, the energy of s_k, is the square of the distance of s_k from the origin,

$$E_1 = \left(\frac{3d}{2}\right)^2 + \left(\frac{3d}{2}\right)^2 = \frac{9}{2}d^2$$

$$E_2 = \left(\frac{3d}{2}\right)^2 + \left(\frac{d}{2}\right)^2 = \frac{5}{2}d^2$$

Similarly,

$$E_3 = \frac{d^2}{2} \qquad \text{and} \qquad E_4 = \frac{5}{2}d^2$$

Hence,

$$\bar{E} = \frac{1}{4}\left[\frac{9}{2}d^2 + \frac{5}{2}d^2 + \frac{d^2}{2} + \frac{5}{2}d^2\right] = \frac{5}{2}d^2$$

and $d^2 = 0.4\bar{E}$. Moreover, for $M = 16$, each symbol carries the information of $\log_2 16 = 4$ bits. Hence, the energy per bit E_b is

$$E_b = \frac{\bar{E}}{4}$$

and

$$\frac{E_b}{\mathcal{N}} = \frac{\bar{E}}{4\mathcal{N}} = \frac{5d^2}{8\mathcal{N}}$$

Hence,

$$P_{eM} = 3Q\left(\frac{d}{\sqrt{2\mathcal{N}}}\right)$$

$$= 3Q\left(\sqrt{\frac{4}{5}\frac{E_b}{\mathcal{N}}}\right) \tag{14.57}$$

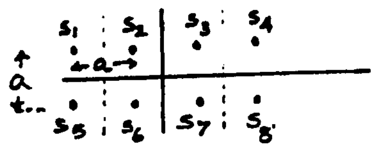
A comparison of this with binary PSK [Eq. (13.21b)] shows that 16-point QAM requires almost 2.5 times as much power as does binary PSK; but the rate of transmission is increased by a factor of $\log_2 M = 4$. This comparison does not take into account the fact that P_b, the BER, is somewhat smaller than P_{eM}. In this case, $N = 2$ and $M = 16$. Hence, the receiver in Fig. 14.8 is preferable. Such a receiver is shown in Fig. 14.12c. Note that because all signals are equiprobable,

$$a_i = \frac{-E_i}{2}$$

For QAM, $\varphi_1(t) = \sqrt{2/T_M}\cos\omega_o(t)$ and $\varphi_2(t) = \sqrt{2/T_M}\sin\omega_o(t)$, where $\omega_o = 2\pi/T_M$.

پاسخ تمرین ۳ :

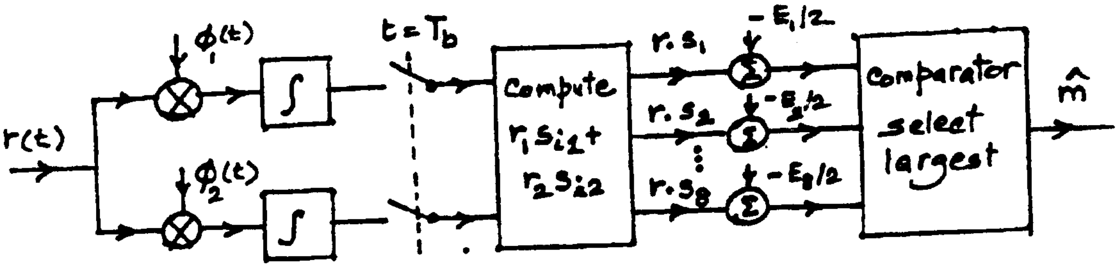
$$P(C|m_1)=P(C|m_4)=P(C|m_5)=P(C|m_8)$$
$$P(C|m_2)=P(C|m_3)=P(C|m_6)=P(C|m_7)$$



$$P(C|m_1)=P\left(n_1<\frac{a}{2},\ n_2>\frac{-a}{2}\right)$$
$$=\left[1-Q\left(\frac{a}{\sqrt{2\mathcal{N}}}\right)\right]\left[1-Q\left(\frac{a}{\sqrt{2\mathcal{N}}}\right)\right]=\left[1-Q\left(\frac{a}{\sqrt{2\mathcal{N}}}\right)\right]^2$$
$$P(C|m_2)=P\left(|n_1|<\frac{a}{2},\ n_2>\frac{-a}{2}\right)$$
$$=\left[1-2Q\left(\frac{a}{\sqrt{2\mathcal{N}}}\right)\right]\left[1-Q\left(\frac{a}{\sqrt{2\mathcal{N}}}\right)\right]$$

and

$$P(C)=\frac{1}{2}\left[P(C|m_1)+P(C|m_2)\right]=\frac{1}{2}\left[1-Q\left(\frac{a}{\sqrt{2\mathcal{N}}}\right)\right]\left[2-3Q\left(\frac{a}{\sqrt{2\mathcal{N}}}\right)\right]$$
$$P_{eM}=1-P(C)=\frac{1}{2}Q\left(\frac{a}{\sqrt{2\mathcal{N}}}\right)\left[5-3Q\left(\frac{a}{\sqrt{2\mathcal{N}}}\right)\right]$$



The average pulse energy $\overline{E}$ is

$$\overline{E}=\frac{1}{8}\left(4\left[\left(\frac{a}{2}\right)^2+\left(\frac{a}{2}\right)^2\right]+4\left[\left(\frac{a}{2}\right)^2+\left(\frac{3a}{2}\right)^2\right]\right)=\frac{3a^2}{2}$$
$$E_b=\frac{\overline{E}}{\log_2 8}=\frac{a^2}{2}$$

and

$$P_{eM}=\frac{1}{2}Q\left(\sqrt{\frac{E_b}{\mathcal{N}}}\right)\left[5-3Q\left(\sqrt{\frac{E_b}{\mathcal{N}}}\right)\right]$$
$$\cong 2.5Q\left(\sqrt{\frac{E_b}{\mathcal{N}}}\right)\qquad \text{assuming } Q\left(\sqrt{\frac{E_b}{\mathcal{N}}}\right)<<1$$

پاسخ تمرین ۴ :

Here

$$\left.\begin{aligned}\phi_1(t) &= \sqrt{\frac{2}{T_M}}\cos\omega_0 t = \sqrt{40}\cos\omega_0 t\\ \phi_2(t) &= \sqrt{40}\sin\omega_0 t\end{aligned}\right\}\ \omega_0 = \frac{2\pi}{T_M}$$

Therefore

$$\begin{array}{l|l} s_1(t) = \sqrt{20}\ \phi_2(t) & s_1 = \sqrt{\ }\ \Phi_2 \\ s_2(t) = \sqrt{5}\ \phi_1(t) & s_2 = \sqrt{5}\ \Phi_1 \\ s_3(t) = -\sqrt{5}\ \phi_1(t) & s_3 = -\sqrt{5}\ \Phi \end{array}$$